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# Atmospheric Energy Transport Over North America For Three Winter Months

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**ABSTRACT**—The excellent North American radiosonde network is used to calculate the poleward energy transport for the continental area during the period January–March 1966. The transport of sensible and latent heat and geopotential and kinetic energy is partitioned according to four circulation modes—mean and transient meridional circulations and stationary and transient eddy circulations. In addition, the roles of various synoptic features in the transient eddy flux are examined.

The mean meridional transport was computed in two ways. One involved a calculation of the contribution of the North American sector to the hemispheric mean meridional transport. Because of strong meridional flow at high levels and a lack of compensating flow at low levels, very large transports were obtained. The transports were much greater than the average for the entire hemisphere and point up the helical structure of the meridional cells. To obtain comparisons with other modes of transport, we made another calculation of the mean meridional transport by subtracting the vertical mean component from the longitudinal average. The results show that the energy transports were large and positive in subtropical latitudes and were zero or small and negative in middle latitudes.

Of the remaining modes, the transient eddy mode was the most effective in transporting energy poleward. The maximum transport occurred at 40°N for both the hemisphere and for North America; however, the value for North America was about 50 percent larger and the latitudinal variation was considerably greater than for the hemisphere. Sensible heat transport was largest, with the maximum latent transport amounting to one-half the sensible heat. Energy fluxes by the standing eddy and transient meridional modes were relatively small.

A brief study of the importance of various large-scale synoptic features in transporting energy indicated that large-amplitude troughs with closed 500-mb Lows are most effective in the transient eddy transport. Indications exist that the largest poleward energy transport is accomplished during the intensifying stage of baroclinic disturbances associated with the 500-mb Lows.

## 1. INTRODUCTION

One of the more prominent features of the atmospheric energy budget is the radiative energy surplus in low latitudes and deficit in high latitudes. Nearly all of the low-latitude surplus is converted into sensible heat, latent heat, and geopotential energy and is transported poleward by various transfer mechanisms in the atmosphere and oceans. In this paper, the characteristics of the energy transport for the North American sector of the Northern Hemisphere are studied for a 3-mo winter period.

Early theories of the general circulation of the atmosphere suggested that the mean meridional circulation was the principal mechanism in the poleward flux of energy. The work of a number of investigators, particularly those associated with the General Circulation Project at the Massachusetts Institute of Technology, has led to the currently accepted view that in middle and high latitudes the poleward transport is accomplished primarily by large-scale eddies. However, it is generally agreed that, equatorward of latitude 30°, the mean meridional cell is the basic transport mechanism, being especially pronounced in the winter season. For discussions of the various mechanisms of the poleward flux of energy, see, for example, Starr and White (1954), Peixoto (1960), Smagorinsky (1964), Holopainen (1965), and Oort (1971*a*).

Since most investigations dealing with energy transports involve estimates based on analyses extending over the entire Northern Hemisphere, certain uncertainties exist due to the sparse observation network over oceanic regions. In the Northern Hemisphere, for example, nearly half of the middle latitudes is covered by oceans, with few radiosonde and rawin stations. Holopainen (1968) found that, over data-sparse oceanic regions in the Northern Hemisphere, large differences exist between atmospheric flow statistics obtained from conventional and objective analyses.

In this paper, the poleward energy transport within the layer between the surface and 100 mb is studied for the months of January, February, and March 1966 for the major portion of the North American continent. By confining the study to this area of the Northern Hemisphere, statistics are obtained that are based on the best observational network available. Since the area covers only a sector of the Northern Hemisphere, the study is limited in that the statistics obtained represent only a contribution to the hemispheric energy transport. However, the values obtained are useful in making comparisons with statistics obtained from studies based on a lower quality hemisphere-wide data network. Such comparisons reveal the significant variations that exist in the modes of the energy transport in different longitudinal sectors.

Specifically, this study examines the transport of each of four energy forms by each of four modes of circulation. In addition, the energy fluxes associated with several types of large-amplitude, upper level wave patterns are briefly examined for some individual synoptic cases.

## 2. MATHEMATICAL FORMULATION OF THE TRANSPORT

Within the atmosphere, energy is transported as sensible heat, latent heat, geopotential energy, and kinetic energy. The total horizontal energy transport ($\Pi_t$) across a given latitude circle between the surface ($p_0$) and 100-mb levels can be written as

$$\Pi_t=\frac{1}{g}\oint\int_{p_0}^{100\text{ mb}}\left(c_pT+\Phi+Lq+\frac{\mathbf{V}\cdot\mathbf{V}}{2}\right)vdpdx \qquad (1)$$

where $T$ denotes temperature, $c_p$ is specific heat at constant pressure, $\Phi$ is geopotential energy per unit mass, $q$ is specific humidity, $L$ is latent heat, $\mathbf{V}$ is the horizontal wind vector, $v$ is the meridional wind component, and $x$ is the distance along a latitude circle.

The sensible heat term is typically the largest of the four energy forms at all levels between 1000 and 100 mb; however, at 100 mb, the geopotential energy term approaches the magnitude of the sensible heat. Near the lower boundary, the latent heat term may be as large as one-tenth of the total energy. Although the kinetic energy is small, its contribution to the poleward transport may be as large as geopotential energy fluxes at high levels for some circulation modes. For estimates of the atmospheric energy content as a function of height and latitude, see Oort (1971*a*).

If we let $E_i$ represent one form of energy per unit mass, then the total energy content per unit mass is

$$E=\sum_{i=1}^{4} E_i \qquad (2)$$

where $i=1, \ldots, 4$ denotes the four energy forms presented in eq (1).

To partition the poleward transport of energy according to circulation modes, we follow Lorenz (1967). Thus, for example, one of the energy forms may be expressed as

$$E_i=\{\overline{E}_i\}+\overline{E}_i^*+\{E_i\}'+E_i'^* \qquad (3)$$

where $\{\}$ is an average with respect to longitude, (*) is a departure from a longitudinal average, $(\overline{\ })$ is a time average, and $(')$ is a deviation from a time average.

Using eq (1) and (3), the poleward flux of a given energy form, $\Pi_i$, averaged with respect to longitude and time is

$$\{\overline{\Pi}_i\}=\frac{1}{g}\int_{p_0}^{100\text{mb}}(\{\overline{E}_i\}\{\overline{v}\}+\{\overline{E}_i^*\overline{v}^*\}+\overline{\{E_i\}'\{v\}'}+\overline{\{E_i'^*v'^*\}})dp\cdot \qquad (4)$$

The total energy transport is obtained by replacing $E_i$ by $E$. The first term on the right side of eq (4) represents the poleward flux by the mean meridional motion over the limited region of study. The second term gives the transport by the standing or stationary eddies (e.g., semipermanent subtropical high-pressure systems and certain planetary waves of middle latitudes). The third term represents the transport by the transient meridional circulation; for example, the expansion and contraction of the circumpolar vortex with respect to time over the limited region of study. The last term represents the transport associated with transient eddies that appear as moving cyclones and anticyclones and transient waves in the westerlies. Throughout this paper, the term "modes" refers to one of the four circulation mechanisms defined in eq (4), whereas the term "synoptic feature" will be used to designate individual synoptic wave patterns.

## 3. DATA AND METHOD OF ANALYSIS

The energy transport calculations were carried out using a spherical grid that covered most of the North American continent (fig. 1). Within this region, there is a total of 90 radiosonde stations. An additional 15 stations outside the region were included for computing the parameters near the boundary. By means of an objective analysis computer program, data from radiosonde stations were extrapolated to a grid with a horizontal resolution of 2.5° latitude-longitude and a vertical resolution of 50 mb between the surface and 100 mb. The effects of variation in surface topography and surface pressure at each radiosonde station were taken into account by weighting the transport in the lowest layer according to the difference between the surface pressure and the pressure at the top of the 50-mb interval above the surface. Using the above grid, the minimum detectable scale is approximately 5°. The program analyzed five parameters: the $u$- and $v$-wind components, temperature, relative humidity, and geopotential height. The details of the analysis are described by Astling (1970). The basic concept of the objective analysis technique was similar to one described by Cressman (1959).

The meridional fluxes of the four energy forms by each circulation mode were calculated for 0000 and 1200 GMT each day. The values were averaged with respect to time and longitude to produce 1- and 3-mo averages. In addition, daily values of transient eddy fluxes at individual gridpoints on one lower (700-mb) and one upper level (250-mb) pressure surface were studied for use in some synoptic case studies.

The months of January, February, and March 1966 were selected because several distinct large-scale circulation features developed or moved over North America during this period, permitting an examination of the energy transport associated with several types of large-scale atmospheric patterns. For example, during March, two situations occurred where a deep closed Low at upper levels developed over the central United States. In the other 2 mo, large-amplitude trough and ridge systems were frequently present. In addition, the classical pattern of an index cycle took place over the western half of the Northern Hemisphere in January and Februray 1966 (Posey 1966, Green 1966).

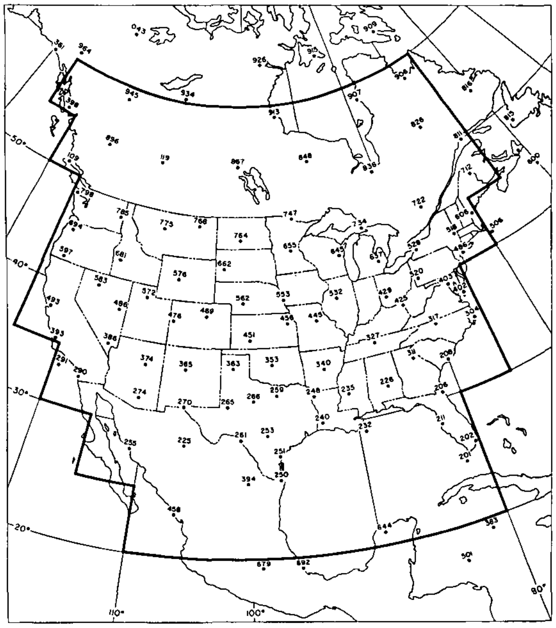

FIGURE 1.—The data region (outlined by a heavy line) and the location of radiosonde stations used in this study.

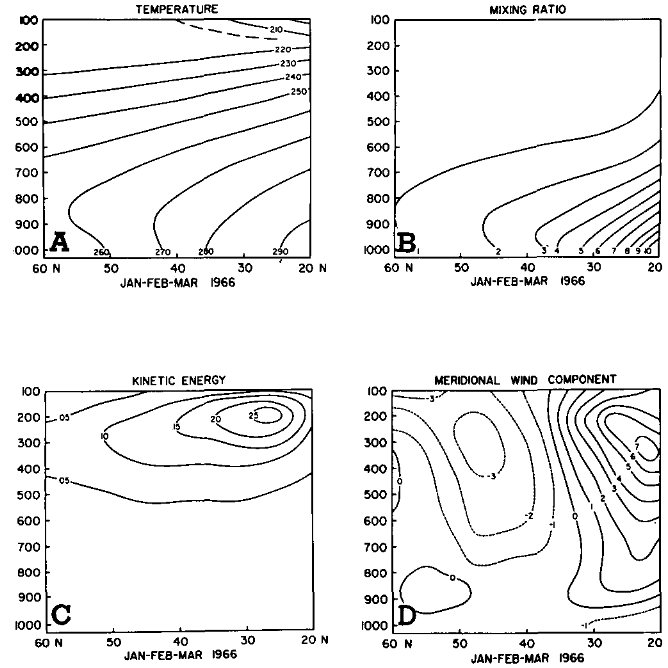


FIGURE 2.—Time- and longitude-averaged values, (A) temperature (°K), (B) mixing ratio (g/kg), (C) kinetic energy (cal/g), and (D) meridional wind component (m/s) for January, February, and March 1966.

## 4. AVERAGES OF THE POLEWARD ENERGY TRANSPORT

Before discussing the poleward energy transport by the four circulation modes, it is helpful to note the general character of the circulation patterns and distribution of the energy parameters that existed over the North American region during the period of study.

The time- and longitude-averaged values of the four parameters used in calculating energy fluxes are shown in figure 2. The distribution of temperature is very similar to the temperature distribution for the entire Northern Hemisphere (fig. 2A) For example, at most latitudes, the temperatures obtained here for North America were within 2°C of the hemispheric averages obtained by Starr and Wallace (1964). The north–south temperature difference at 1000 mb is 35°C between latitudes 20° and 60°N, while at 100 mb the gradient is reversed and amounts to 15°C.

Figure 2B reveals, as expected, that the moisture is concentrated below the 700-mb level with the largest amounts at 20°N. The horizontal gradient of moisture and, thus, latent heat is very large compared to the gradient of sensible heat. Obviously, surface cyclones and anticyclones, as well as surface terrain features, strongly influence the moisture transport.

The vertical distribution of kinetic energy for January, February, and March 1966 (fig. 2C) indicates that the average position of the jet stream was located near 27°N during the data period. This position is somewhat south of the climatic mean 300-mb position of the jet stream over this region as indicated by Lahey et al. (1960).

Figure 2D shows that, for the 3-mo period, the mean meridional wind component was northerly (negative) throughout the troposphere and lower stratosphere poleward of 32°N. The mean circulation over North America during this time was characterized by a trough over the eastern portion of the area with a ridge in the west producing the northerly flow (Posey 1966, Green 1966, Stark 1966). Comparisons with the mean surface resultant winds prepared by Bryson (1966) indicate that the region of confluence associated with the Pacific front was displaced south of its mean position over the midwestern United States during the period of study. The mean meridional wind component equatorward of 32°N was characterized by strong southerly flow in the upper troposphere and weak northerly flow near the surface. Although the equatorward moving branch was not well developed, the pattern, in general, is that associated with the Hadley cell. Except for the intensity of the flow, the pattern presented in figure 2D is in qualitative agreement with the time-averaged pattern obtained by Palmén and Vuorela (1963).

### a. Mean Meridional Circulation

The vertical and latitudinal distributions of the energy transports by the mean meridional circulation for the North American sector as calculated from eq (4) are presented in figure 3. Equatorward of about 32°N, the net transport of both sensible heat and geopotential energy for the surface to 100-mb layer was toward the north. At these latitudes, the transport of *both* energy forms by the high-level northward moving branch of the Hadley circulation was much greater than that accomplished by the low-level southward moving branch (figs. 3A, 3D). Poleward of 32°N over the North American sector, the transports of sensible heat and geopotential energy were mainly southward, with the high-level south-

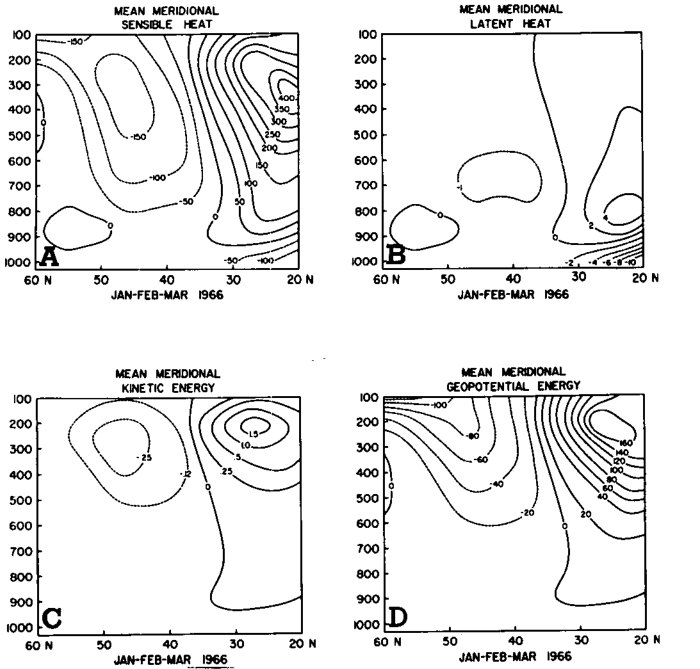


FIGURE 3.—Average poleward flux of energy by the mean meridional circulation over North America for January, February, and March 1966 of (A) sensible heat, (B) latent heat, (C) kinetic energy, and (D) geopotential energy. Units are $10^2$ cal·s$^{-1}$·mb$^{-1}$·cm$^{-1}$.

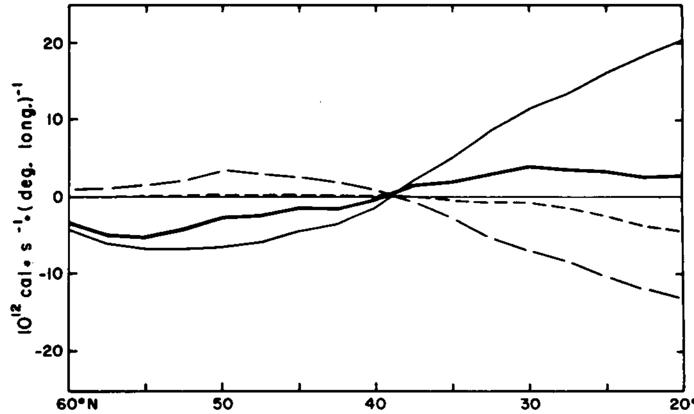


FIGURE 4.—Vertically integrated transports by the mean meridional circulation of geopotential energy (thin solid line), sensible heat (long dashed line), latent heat (short dashed line), and their sum (heavy solid line). Units are $10^{12}$ cal ·s$^{-1}$·(deg. long.)$^{-1}$.

ward moving branch of the Ferrel circulation being dominant.

In general, the transports by the mean meridional circulation for the North American sector were much larger than those calculated by Holopainen (1965) and Oort (1971*a*), being nearly two orders of magnitude larger at some latitudes. The large values were principally the result of strong meridional wind speeds and a tendency for the existence of very weak compensating flow between high and low elevations over North America.

Because of the large transport estimates, the question might be raised as to whether a mean meridional transport calculated in this way for a limited area has significance. The authors feel that it does, for it provides a quantitative estimate of the large longitudinal variations that exist in the intensity of the meridional circulations. Moreover, the insufficient compensating motion in the mean meridional circulation over North America emphasizes the basic helical nature of meridional overturnings. The longitudinal variations do not consist simply of more intense overturnings in a north–south plane at some longitudes than others, but rather represent large poleward transports at high levels for some longitudes with the tendency for low-level compensating motion to occur at other longitudes.

Although the mean meridional transport estimates, as calculated from eq (4), point out some of the often neglected characteristics of the north–south overturnings, the large values make comparisons with the estimates obtained for the other three modes of circulation difficult. Consequently, a second method was used to calculate the mean meridional fluxes for the North American sector following a suggestion by Oort (1971*b*). The vertical mean component of the energy terms and meridional wind components were subtracted from the computed flux values according to the following:

$$\frac{1}{g}\int_{p_0}^{100\text{ mb}} \{\overline{E}\}''\{\overline{v}\}''dp \qquad (5)$$

where

$$\{E\}''=\{E\}-\int_{p_0}^{100\text{ mb}} \{E\}dp/(p_0-100). \qquad (6)$$

A similar expression was used for $\{v\}''$.

With this new definition, the transports calculated for the mean meridional circulation were two orders of magnitude smaller than the estimates obtained from $\{\overline{E}\}\{\overline{v}\}$. From the results illustrated in figure 4, one can see that the net transport was northward between 20°N and approximately 38°N latitude and amounted to approximately $3\times10^{12}$ cal·s$^{-1}$·(deg. long.)$^{-1}$. In this region, the geopotential and sensible heat terms were dominant, with northward transport by the $\Phi$ term partially canceling the southward flow of sensible heat. In contrast, north of 38°N, each energy term contributed much less to the mean meridional fluxes, and the net transport was directed southward. These estimated transports are larger than the transports by eddy circulations in southern latitudes (discussed in the next section) but are smaller in middle latitudes and agree with the results obtained by Oort (1971*a*) and Holopainen (1965).

## b. Transient Eddies

The results of the energy transport calculations of the transient eddy, standing eddy, and transient meridional circulations are listed in table 1. At all latitudes but 20°N, the transient eddy circulation was the dominant transport mechanism among the remaining circulation modes, with the sensible and latent heat fluxes being dominant. Poleward fluxes were largest in the middle latitudes, where the transient eddy mode contributed approximately three-fourths of the total poleward transport by the modes in-

TABLE 1.—*Average transport of sensible and latent heat and geopotential and kinetic energy over North America for January, February, and March 1966 by transient eddies, standing eddies, and transient meridional circulation. Values represent the poleward transport across latitudes at 5° intervals between the surface and 100 mb. Units are $10^{12}$ cal · $s^{-1}$ · (deg. long.)$^{-1}$.*

| Mode/Latitude | 60°N | 55° | 50° | 45° | 40° | 35° | 30° | 25° | 20°N |
|---|---|---|---|---|---|---|---|---|---|
| Transient eddy | | | | | | | | | |
| $c_pT$ | 0. 569 | 0. 736 | 1. 292 | 1. 794 | 2. 103 | 1. 933 | 1. 286 | 0. 525 | 0. 008 |
| $Lq$ | . 119 | . 214 | 0. 356 | 0. 506 | 0. 628 | 0. 922 | 1. 047 | . 936 | . 339 |
| $\phi$ | —. 039 | —. 125 | —. 072 | . 042 | . 053 | . 031 | —0. 047 | —. 122 | —. 069 |
| (**V**•**V**)/2 | —. 011 | —. 047 | —. 031 | —. 003 | . 050 | . 083 | . 111 | . 142 | . 067 |
| Total | 0. 638 | 0. 778 | 1. 545 | 2. 339 | 2. 834 | 2. 969 | 2. 337 | 1. 481 | 0. 345 |
| Standing eddy | | | | | | | | | |
| $c_pT$ | —0. 164 | 0. 339 | 0. 531 | 0. 386 | 0. 153 | 0. 094 | 0. 378 | 0. 089 | —0. 030 |
| $Lq$ | . 028 | . 169 | . 222 | . 100 | . 028 | . 081 | . 206 | . 197 | —. 292 |
| $\phi$ | . 028 | —. 056 | —. 089 | —. 044 | —. 081 | —. 017 | —. 136 | . 044 | . 039 |
| (**V**•**V**)/2 | —. 008 | —. 008 | —. 003 | . 017 | . 053 | . 097 | . 080 | . 022 | . 006 |
| Total | —0. 172 | 0. 444 | 0. 661 | 0. 459 | 0. 153 | 0. 252 | 0. 809 | 0. 352 | —0. 277 |
| Transient meridional | | | | | | | | | |
| $c_pT$ | 0. 269 | 0. 342 | 0. 133 | 0. 100 | 0. 361 | 0. 317 | 0. 378 | 0. 364 | 0. 194 |
| $Lq$ | . 094 | . 050 | . 039 | . 081 | . 139 | . 156 | . 342 | . 297 | . 214 |
| $\phi$ | . 047 | —. 008 | —. 067 | —. 044 | . 003 | —. 014 | —. 036 | —. 025 | . 028 |
| (**V**•**V**)/2 | —. 011 | —. 019 | . 000 | . 031 | . 042 | . 039 | . 047 | . 047 | . 019 |
| Total | 0. 399 | 0. 365 | 0. 105 | 0. 168 | 0. 545 | 0. 498 | 0. 731 | 0. 683 | 0. 455 |
| Grand total | 0. 865 | 1. 587 | 2. 311 | 2. 966 | 3. 532 | 3. 719 | 3. 877 | 2. 516 | 0. 523 |

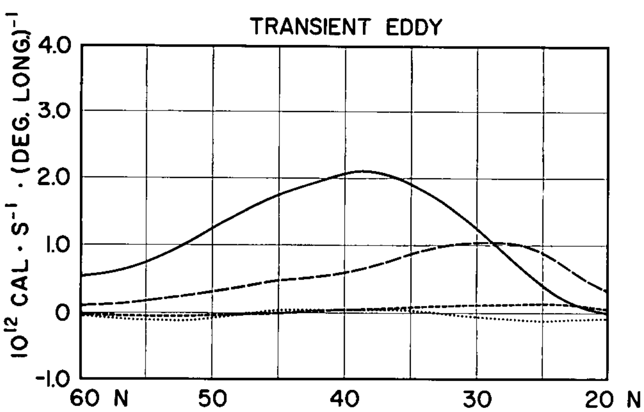


FIGURE 5.—Mean poleward flux of sensible heat (solid line), latent heat (long dashed line), kinetic energy (short dashed line), and geopotential energy (dotted line) over North America by transient eddy circulations for January through March 1966. Units are $10^{12}$ cal ·$s^{-1}$·(deg. long.)$^{-1}$.

volving transient and eddy circulations. In figure 5, the transient eddy fluxes for the four energy forms are presented graphically. The flux of sensible heat reached a maximum of $2.2\times10^{12}$ cal · $s^{-1}$ · (deg. long.)$^{-1}$ at 37.5°N. On the other hand, the flux of latent heat was largest south of this latitude, reaching a value of $1.0\times10^{12}$ cal · $s^{-1}$ · (deg. long.)$^{-1}$ at 30°N. The difference between the latitudes of maximum flux of sensible heat and latent heat is likely associated with the occurrence of cyclones along the polar front. In North America for the winter season, the mean latitude of the polar front is between 35° and 40°N, and it is at these latitudes that the sensible heat flux is a maximum. The moisture source for cyclones over the eastern one-half or two-thirds of the continent is the Gulf of Mexico, Caribbean Sea, and warm Atlantic adjacent to the Southeastern States. As moist air moves northward ahead of the migratory (and often occluding) cyclones and dry air moves southward to the rear, a maximum gradient of moisture and flux is typically found to the south of the cyclone track. The vertical distribution of the average poleward energy flux by transient eddies for January through March 1966 is shown in figure 6 for each of the energy terms.

Note that near the tropopause in low latitudes the kinetic energy fluxes were the largest of the four transport energy terms, exceeding $0.6\times10^2$ cal·$s^{-1}$·$mb^{-1}$·$cm^{-1}$ at 25°N near 200 mb. Kinetic energy fluxes for individual months were a maximum at upper levels in the vicinity of the jet stream where the kinetic energy was largest. In February, for example, the poleward fluxes were largest at the tropopause level between 20° and 40°N, where the winds were much stronger than in either January or March 1966. In studies of the poleward transport of energy at high levels, the kinetic energy flux should not be ignored.

## c. Standing Eddies

An examination of figure 7, which depicts the vertical and latitudinal variations of the standing eddy fluxes, reveals that near 50°N fluxes of sensible and latent heat amount to 2 and $1.25\times10^2$ cal·$cm^{-1}$·$s^{-1}$·$mb^{-1}$, respectively. The relatively large fluxes at 50°N may be the result of a strong, standing eddy circulation associated

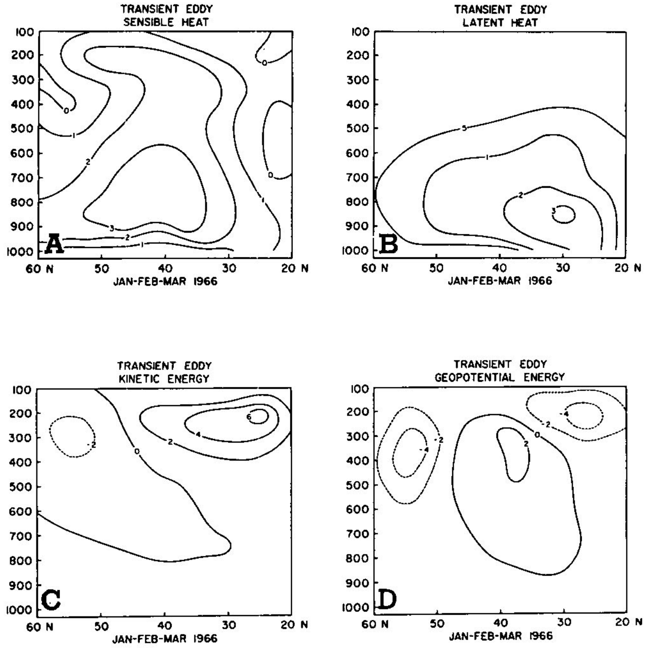


FIGURE 6.—Time- and longitude-averaged poleward energy transport of (A) sensible heat, (B) latent heat, (C) kinetic energy, and (D) geopotential energy by transient eddies. Units are $10^2$ cal $\cdot s^{-1} \cdot mb^{-1} \cdot cm^{-1}$.

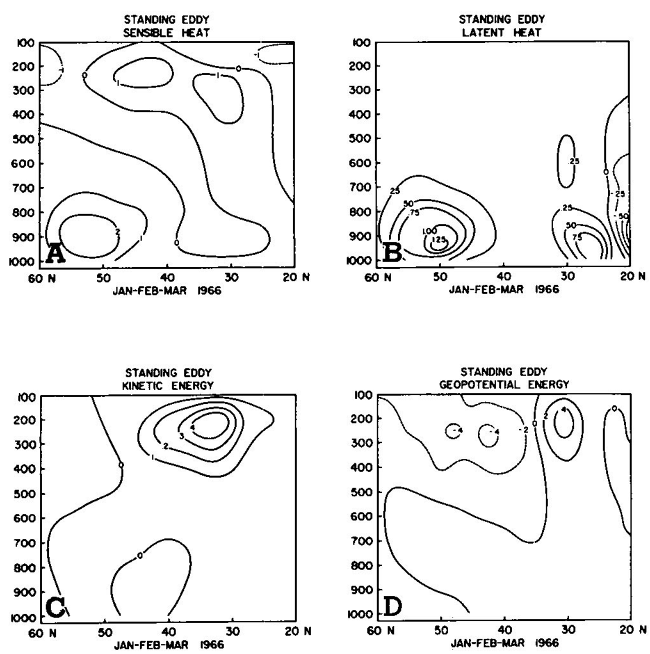


FIGURE 7.—Same as figure 6 for transport by standing eddies.

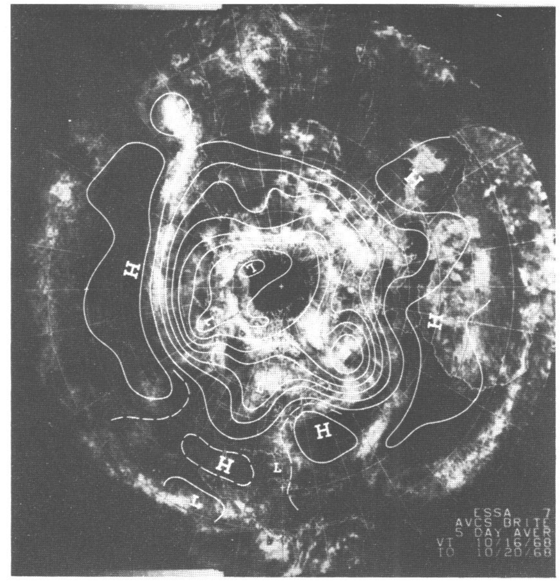


FIGURE 8.—Five-day (Oct. 16–20, 1968) average cloud brightness picture from ESSA 7 with the corresponding 5-day mean 700-mb height contours superimposed (Andrews 1969).

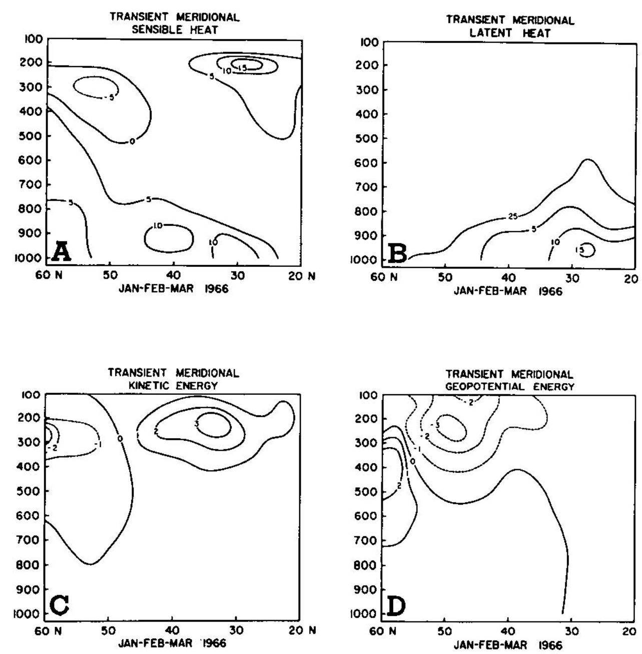


FIGURE 9.—Same as figure 6 for transport by transient meridional circulations.

with the Aleutian Low. A strong southwesterly flow of relatively warm, moist air enters the region along the Northwest Pacific Coast of the continent. The secondary maximum of standing eddy latent heat flux noted at latitudes 25° to 30°N is accomplished primarily by quasi-stationary subtropical anticyclones. An example of this circulation mode is illustrated in figure 8, which shows a 5-day mean 700-mb height field superimposed on an ESSA 8 cloud brightness map of the Northern Hemisphere for Oct. 16–20, 1968. The extensive areas of cloudiness present in the flow around the west side of the subtropical high are especially evident between 25° and 30°N over the central Atlantic and eastern United States.

The standing eddy flux of latent heat at 20°N was the only circulation mode of the three shown in figures 6B, 7B, and 9B in which the net transport of latent heat was directed equatorward. For this circulation mode, an apparent region of moisture divergence was present near 22°N with southward transports on the equatorward side and northward transports on the poleward side. Adem (1968), in a study of the atmospheric water budget for the whole Northern Hemisphere for one winter season, found that standing eddy fluxes of moisture were large and directed equatorward at low latitudes. Hastenrath

TABLE 2.—*Comparison of the average transient eddy flux of total energy as calculated by Oort (1971a) for the 3 mo of January, February, and March (5 yr of data) with the equivalent flux calculated for the North American sector for January–March 1966. Oort's figures are for the surface to 75 mb; those for North America are for the surface to 100 mb. Units are $10^{12}$ cal·s$^{-1}$·(deg. long.)$^{-1}$.*

| | 60°N | 55° | 50° | 45° | 40° | 35° | 30° | 25° | 20°N |
|---|---|---|---|---|---|---|---|---|---|
| Northern Hemisphere (Oort) | 1. 200 | 1. 425 | 1. 597 | 1. 843 | 2. 122 | 2. 326 | 2. 079 | 1. 415 | 0. 652 |
| North America | 1. 037 | 1. 143 | 1. 650 | 2. 507 | 3. 279 | 3. 467 | 3. 068 | 2. 164 | 0. 800 |

(1966) found an export of latent heat from the Gulf of Mexico.

The standing eddy flux of kinetic energy in figure 7C is similar to the flux by transient eddies with maximum values near 200 mb and 32°N (fig. 6C). The large poleward flux values are probably associated with the large-scale trough over the United States. An examination of standing eddy fluxes of kinetic energy at individual gridpoints indicated that the maximum poleward fluxes occurred along the northern edge of the jet stream which was located over Mexico and the southeastern United States. The export of kinetic energy from the jet core maximum by the standing eddy flux is not characteristic of the concept of eddies supplying energy to the mean flow.

The standing eddy flux of geopotential energy (fig. 7D) is largest in the levels above 300 mb with values similar to the transient eddy flux of geopotential energy. Although the geopotential energy fluxes are small, they indicate the presence of nongeostrophic motions since no geopotential energy flux would exist if the flow was geostrophic. As pointed out by Godson (1950), the difference between the actual and geostrophic wind speed is largest for speeds exceeding 25 m/s. Consequently, large fluxes of geopotential energy should be expected at higher levels. Positive fluxes would be associated with subgeostrophic winds in southward flow and supergeostrophic winds in northward flow of a stationary eddy.

### d. Transient Meridional Circulations

An examination of table 1 reveals that the total energy fluxes accomplished by the transient meridional circulation were largest at 25°–30°N. This latitudinal region is the transition zone between the Hadley circulation in the tropical latitudes and the circumpolar vortex in middle and polar latitudes. It is likely that the expansion and contraction of the meridional circulation system would produce the large transient fluxes in this latitude zone. In figure 9, the fluxes of sensible heat and kinetic energy at high levels and latent heat at low levels appear to be responsible for the maximum at 25° and 30°N.

### e. Comparisons With Other Studies

An advantage in studying the energy transport for a limited longitudinal sector is that statistics obtained for the sector can be compared with statistics based on hemispheric wide studies to describe significant longitudinal variations in the modes of the transport. In this section, we turn to a comparison of the circulations involving space and time deviations obtained for North America with the excellent results obtained by Oort (1971*a*) for an area completely encircling the Northern Hemisphere. In making the comparison, it is important to note that Oort employed the following partitioning:

$$\{\overline{\Pi}_i\}=\{\overline{E}_i\}\{\bar{v}\}+\{\overline{E}_i^*\bar{v}^*\}+\{\overline{E_i'v'}\}, \tag{7}$$

while in this study,

$$\{\overline{\Pi}_i\}=\{\overline{E}_i\}\{\bar{v}\}+\{\overline{E}_i^*\bar{v}^*\}+\overline{\{E_i\}'\{v\}'}+\{\overline{E_i'^*v'^*}\} \tag{8}$$

where $\{\overline{\Pi}_i\}$ is the time- and longitude-averaged transport of one energy form for a 1-mb layer. Thus the last term in eq (7), which Oort labels the "transient eddy" mode, is equivalent to the sum of the transient eddy and transient meridional modes calculated here; that is,

$$\{\overline{E_i'v'}\}=\overline{\{E_i\}'\{v\}'}+\{\overline{E_i'^*v'^*}\}. \tag{9}$$

Equation (9) has been used to equate the transient eddy term calculated by Oort (1971*a*) with the equivalent estimates obtained in this study (table 2). Oort's January, February, and March mean monthly values (based on 5 yr of data) for the Northern Hemisphere have been averaged to provide a comparison with the means for the 3-mo period (January–March 1966) of this study. Both sets of data are for the total energy and are expressed in units of $10^{12}$ cal·s$^{-1}$· (deg. long.)$^{-1}$. One should remember that Oort's figures represent a 5-yr average, while those in this study involve only 1 yr.

Since the transient eddy mode involves synoptic scale size features, the truncations that result from using a limited longitudinal sector are not severe. In table 2 there is good agreement between the statistics for the Northern Hemisphere and those for North America. In both sets, the maximum transport occurs at 35°N but with the North American sector having a 50-percent greater value. Since the North American area tends to be a region with vigorous cyclone development and since the early part of 1966 was a period of unusual cyclonic activity, the larger figure was expected. It is interesting to note that the latitudinal variation of flux is distinctly larger over North America than for the hemisphere. In fact, at 55°–60°N, the hemispheric values exceed those for North America. Over North America, the mean winter position of the jet stream, and the associated storm track, is displaced somewhat farther south than in the eastern Atlantic, Eurpoean, and eastern Pacific sectors. Thus, if one does hemispheric averaging, the flux of energy by transient features will show a smoother latitudinal profile than for a single sector.

TABLE 3.—*Same as table 2 for the standing eddy mode*

| | 60°N | 55° | 50° | 45° | 40° | 35° | 30° | 25° | 20°N |
|---|---|---|---|---|---|---|---|---|---|
| Northern Hemisphere (Oort) | 1.083 | 1.522 | 1.704 | 1.458 | 0.943 | 0.579 | 0.386 | 0.300 | 0.214 |
| North America | −0.172 | 0.444 | 0.661 | 0.459 | 0.153 | 0.256 | 0.809 | 0.352 | −0.277 |

An examination of a breakdown of the total energy flux by the transient eddy mode into the four energy forms reveals that, both for North America and the Northern Hemisphere, the sensible heat flux was largest, and the maximum flux was at 40°N. In the case of the latent heat flux, both studies obtained a maximum at 30°N; however, for the Northern Hemisphere, the value at this latitude exceeded the value for sensible heat, while for the continental sector of North America the flux of latent heat was slightly less than that of sensible heat. Over North America, the variation in the latent heat flux between 30° and 60°N amounted to a factor of more than eight, while for the Northern Hemisphere the factor was only about four, which again emphasizes the sharper latitudinal variations in North America. In both studies, the flux of geopotential energy was quite small and generally negative. The flux of kinetic energy was also small in both studies.

Table 3 provides a comparison of the standing eddy flux of the total energy for North America with Oort's values for the Northern Hemisphere. The most noteworthy difference between the two sets of statistics is the considerably greater transport by the standing eddy mode for the Northern Hemisphere latitudes north of 40° or 45°N. The large standing eddies associated with features such as the Aleutian and Icelandic Lows, which are truncated when only the North American sector is studied, appear to be responsible for this greater transport. In looking at the breakdown of the transport into each of the four energy forms (table 4 in Oort's paper and table 1 in this paper), the large transport of sensible heat by the standing eddies averaged around the higher latitudes of the hemisphere stands out. As Oort (1971*a*) points out, the standing eddies rather than the transient eddies appear to accomplish the bulk of the transport of sensible heat at higher latitudes. Although the North American data does not support this observation, the truncations that exist in using the limited sector may well obscure the large transport.

## 5. IMPORTANCE OF VARIOUS LARGE-SCALE SYNOPTIC WEATHER SYSTEMS

Since the transient eddy circulations involve waves of various lengths and shapes, the question arises as to which eddies are most effective in the poleward transport. In calculating the transport associated with various wave numbers, Wiin-Nielsen et al. (1963) found that wave numbers ≤4 were responsible for approximately 45 percent of the sensible heat transport while wave numbers 6–8 accounted for 40 percent of the transport at 40°N, the middle latitude of the area studied here. However, as Barrett (1961) has pointed out, wave shape has been neglected or considered as secondary in importance in the transport. In this paper, the role of wave shape has been investigated for wave numbers 6–9 (i.e., wavelengths between 40° and 60° of long.). Because of the limited longitudinal sector, smaller wave numbers could not be analyzed; however, from baroclinic instability theory, wave numbers around 6 are quite active and are thus of interest.

### a. Selection of Synoptic Features

In examining the eddy transport associated with wavelengths between 40° and 60° of longitude, one encounters the problem of which pressure level should be used in defining the wave pattern. The 500-mb surface was chosen as being the most representative, but the transports were calculated for the 700- and 250-mb levels. Winston (1961) found that the energy transport for the 850- to 500-mb layer is well correlated with the transport for the entire troposphere. The 700-mb surface was chosen as representative of the 850- to 500-mb layer while the transport at 250 mb was chosen to represent the high levels. Because of the relative importance of the various energy terms, only the sensible heat and latent heat are discussed for the 700-mb surface, the discussion of the fluxes at 250 mb are confined to sensible heat, geopotential energy, and kinetic energy.

For the 3-mo period of study, a total of 20 cases were selected and divided into five types of synoptic features based on the 500-mb geopotential height field. Included were two types of large-scale trough systems, a closed Low, a large-scale ridge, and a zonal flow pattern, all of which are characteristic of middle latitude synoptic weather systems. The five synoptic features and the criteria used in selecting them were:

1. Closed Low—an upper level Low center in which there were two or more closed contours in the 500-mb height field.
2. Open wave trough—a trough in which the wave amplitude (defined as the distance between the northern and the southernmost positions of a given contour line) of the 500-mb height contours was greater than 15° of latitude and the trough axis intersected the central meridian by an angle of less than 10°.
3. Eastward tilting trough—a trough in which the amplitude of the 500-mb height contours was greater than 15° of latitude and the trough axis intersected the central meridian by an angle greater than 20°.
4. Ridge—a ridge in which the wave amplitude of the 500-mb height contours was greater than 15° of latitude and the ridge axis intersected the central meridian by an angle of less than 10°.
5. Zonal flow—the contour pattern over at least a 10° latitude zone was nearly parallel to the latitude circles, any wave present having an amplitude of less than 5° latitude.

Examples of the types of cases used in this study are shown in figure 10. Five cases satisfied the criteria for

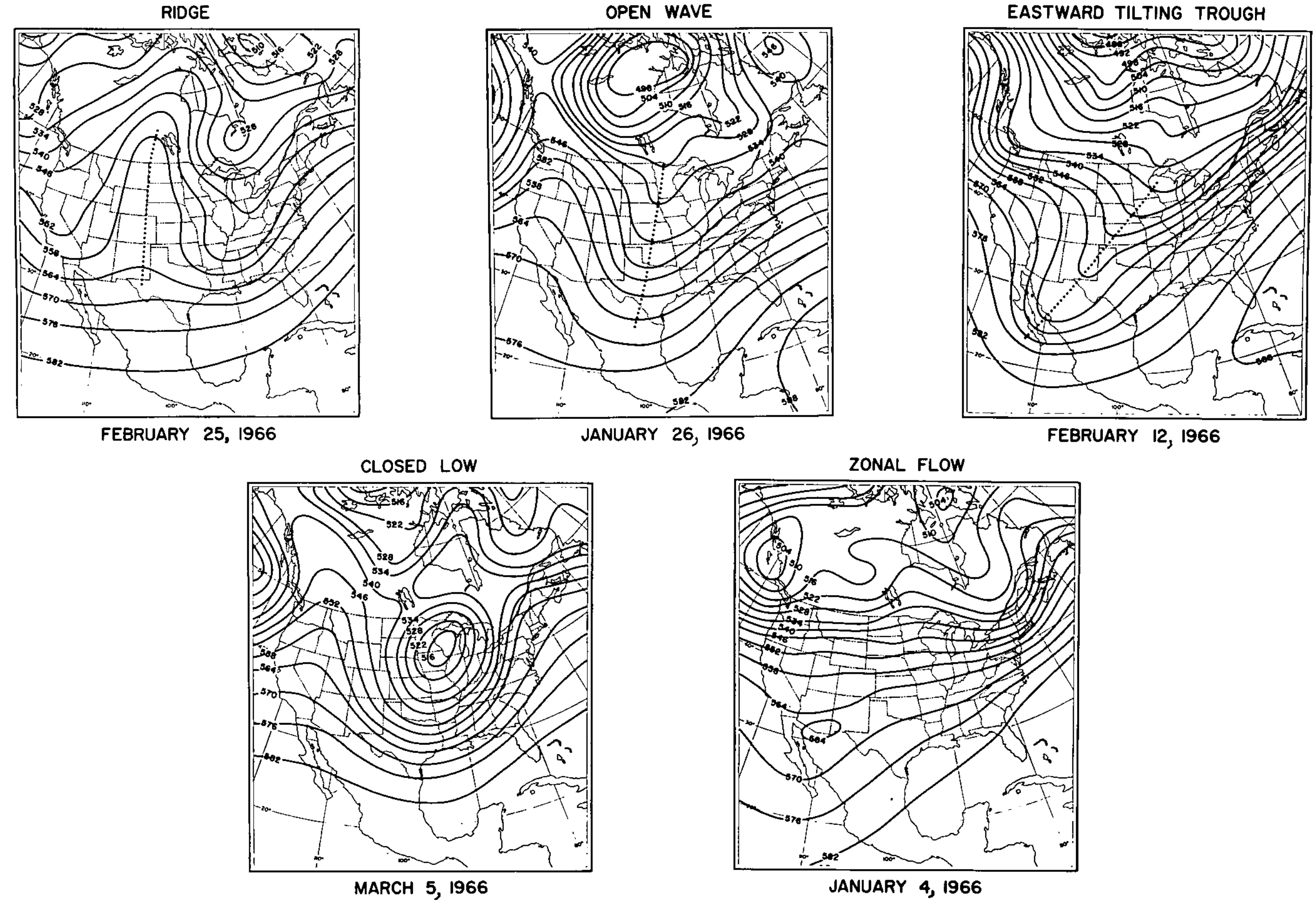


FIGURE 10.—Examples of the five synoptic features listed in table 5. Solid lines are 500-mb contours and are labeled in tens of meters. Dotted lines represent trough and ridge axes.

each of the first three types. Two cases satisfied the criteria for the ridge type, and three cases fulfilled the zonal flow criteria for a portion of the region of study. All of the selected cases occurred on nonconsecutive days. The first three synoptic features had wavelengths of approximately 40°–60° of longitude (i.e., wave no. 6–9) while the cases that satisfied the fourth had wavelengths of approximately 40° of longitude.

## b. Composite Fluxes

For the open waves and eastward tilting troughs, composite averages of the eddy fluxes were obtained at 2.5° grid intervals using the trough axis as a reference line. Similarily, composite averages were obtained for the closed Low circulation type except that the Low center was used as the reference point.

Composite 700- and 250-mb averages for the latitude of maximum transport for each of the synoptic features are listed in table 4. At both elevations, the closed Low is decidedly the most effective feature in the poleward transport. The energy transport associated with the open wave is only about one-half that produced by the closed Low. The eastward tilting trough, which is an essential mechanism in the poleward transport of eastward momentum, is quite ineffective in the poleward transport of energy, at least in the small sample of cases studied here. Except for a sizable transport of sensible heat at 250 mb, the ridge and zonal flow features contributed little to the transport.

TABLE 4.—*Composite average of the poleward flux of energy at the latitude of maximum transport accomplished by the five synoptic features. Units are $10^2$ cal·s$^{-1}$·mb$^{-1}$·cm$^{-1}$.*

| Feature | 700 mb | | 250 mb | | |
|---|---|---|---|---|---|
| | $c_pT$ | $Lq$ | $c_pT$ | $\phi$ | $V^2/2$ |
| Closed Low | 12.3 | 4.7 | 7.9 | 2.2 | 0.7 |
| Open wave | 6.6 | 2.1 | 5.1 | −0.9 | 1.0 |
| Eastward tilting trough | 0.4 | 1.2 | −2.9 | −1.2 | 1.3 |
| Ridge | −1.4 | 1.0 | 9.0 | −1.7 | −0.2 |
| Zonal flow | 1.4 | 0.5 | 0.8 | −0.3 | 0.6 |

The composite energy flux averages accomplished by the closed Low are shown in figure 11. The Low center is represented by the intersection of the coordinate axes, and the grid spacing from the center is marked for every 5°. At 700 mb, an area of large transport of sensible heat was located to the east of the Low and another to the west, with the maximum value of $30\times10^2$ cal·s$^{-1}$·mb$^{-1}$·cm$^{-1}$ located between 5° and 10° west of the Low. Similar sensible heat flux values were found at 250 mb; however, because of the westward tilt of Lows with respect to height, the maximum flux of sensible heat at 250 mb was displaced westward. In contrast, the flux of latent heat

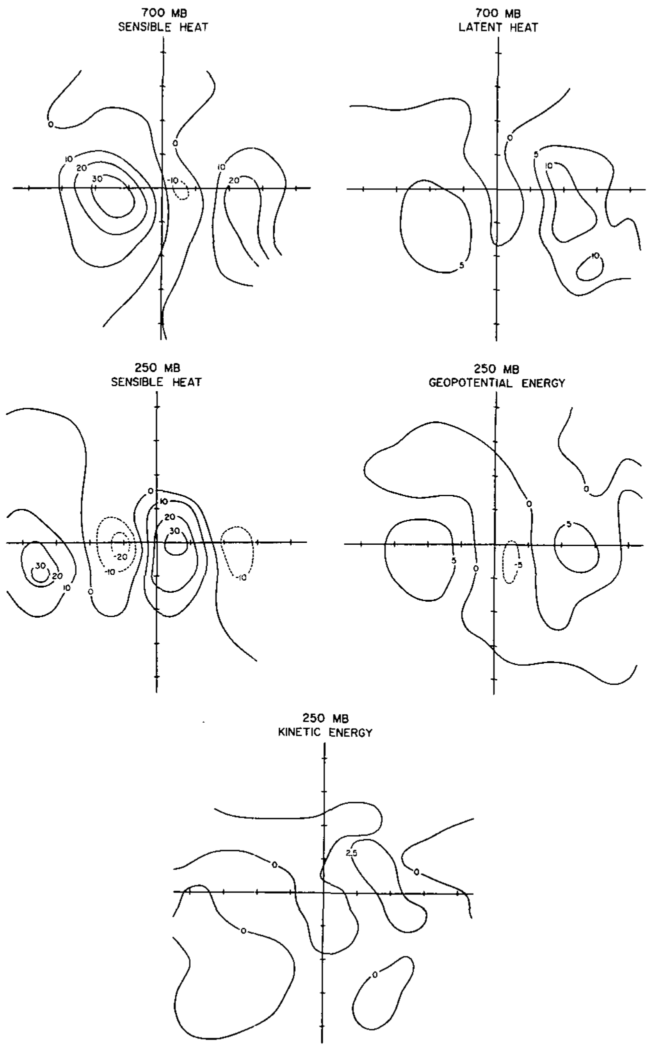


FIGURE 11.—Composite averages of the energy fluxes accomplished by the closed Low synoptic features at 700 and 250 mb. Distances from the Low center are marked at grid intervals of 5°. Units are $10^2$ cal $\cdot s^{-1} \cdot mb^{-1} \cdot cm^{-1}$.

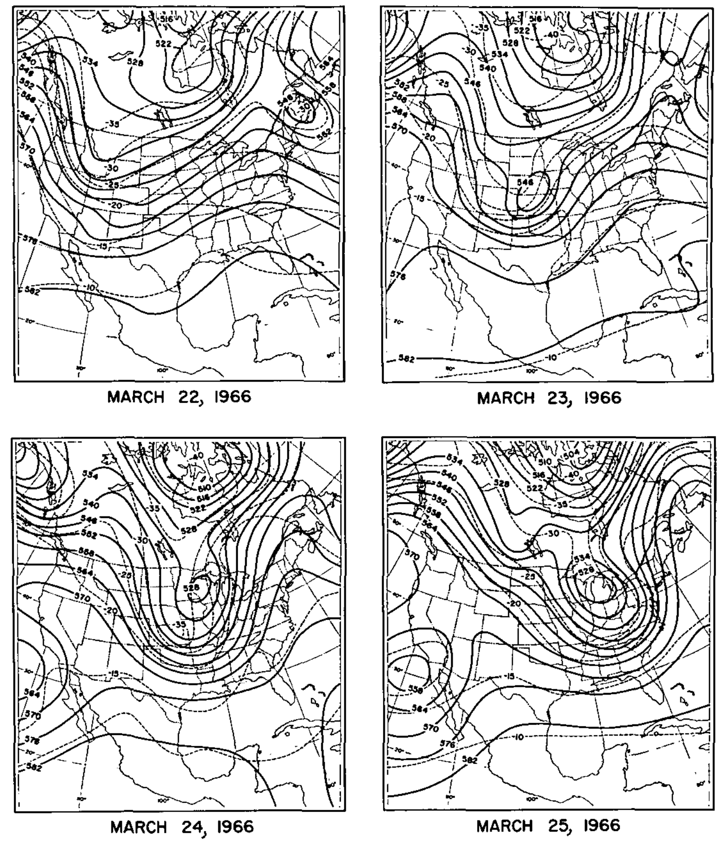


FIGURE 12.—The 500-mb charts for 0000 GMT on Mar. 22–25, 1966.

was large only to the east of the Low, with values of $10 \times 10^2$ cal$\cdot s^{-1} \cdot mb^{-1} \cdot cm^{-1}$ at 700 mb. Small flux values of geopotential and kinetic energy were present at 250 mb. In all cases, the areas of maximum fluxes were separated by an elongated area of small negative fluxes that extended north–south through the Low center.

The significance of the energy fluxes accomplished by closed Low synoptic features is evident when a comparison is made with the average fluxes by transient eddies in middle latitudes as shown in figure 6. The maximum fluxes of the four energy forms across the latitude that intersects the closed Low center are nearly 10 times as large as the longitude-averaged poleward energy transport at both 700 and 250 mb. Moreover, the average energy flux across the center latitude of the closed Low is approximately five times larger than the transient eddy flux shown in figure 6. If we assume that the 3-mo averaged fluxes by transient eddies are representative of the hemisphere as a whole, two intense closed Lows present in middle latitudes could account for the poleward fluxes by transient eddy circulations. Because the transient flux (per deg. of long.) over North America very likely exceeded that for the whole hemisphere, the effectiveness of the transport by the closed Low feature is all the more impressive. The importance of closed Lows in transporting energy is not surprising when one notes that Palmén (1959), for example, estimated that a closed Low associated with a well-developed extratropical cyclone produced enough kinetic energy to balance the frictional dissipation for the entire Northern Hemisphere poleward of latitude 30°N.

## c. A Case Study

Because of the large energy fluxes accomplished by closed Lows, a more detailed study was made of the fluxes associated with the evolution of one such system. Figure 12 presents a sequence of 500-mb charts for the period Mar. 22–25, 1966. The system began to develop early on March 22 when a trough system moved from the Pacific into the western United States. Within 24 hr, the trough had deepened and a closed 500-mb Low appeared over Nebraska. During the next 2 days, the closed Low intensified as it moved eastward.

The daily transient eddy fluxes of sensible and latent heat at 700 mb are presented in figures 13 and 14. During the first day, the flux of sensible heat was moderate. It increased on the second day, reached a maximum on the third day, and decreased slightly on the fourth day. Although the flux of latent heat was less, it did reach 40 percent of the sensible heat flux on the last 2 days when the system had *moved far enough to the east to tap the*

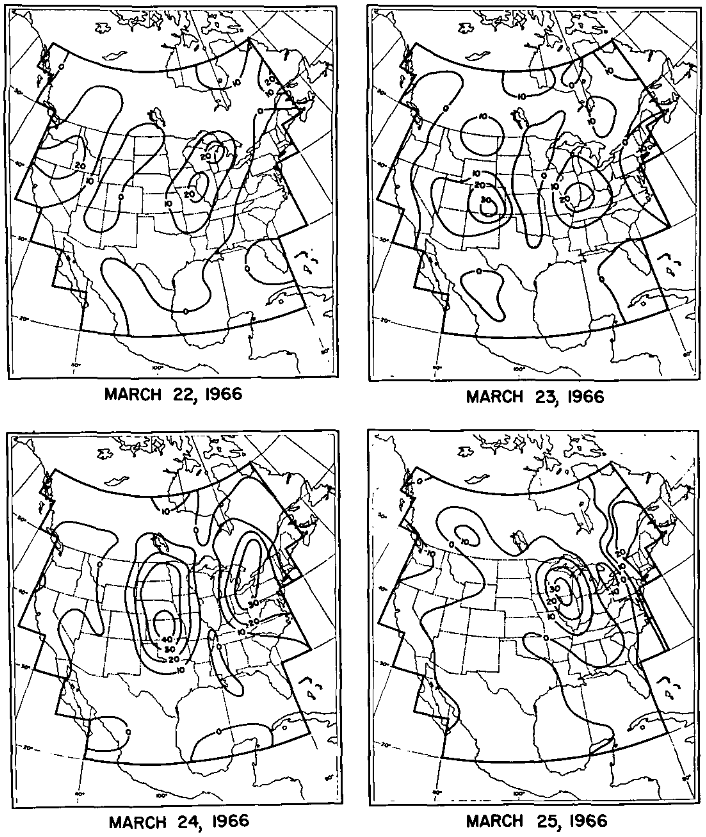


FIGURE 13.—Transient eddy fluxes of sensible heat at 700 mb for 0000 GMT on Mar. 22–25, 1966. Units are $10^{12}$ cal $\cdot s^{-1} \cdot mb^{-1} \cdot cm^{-1}$.

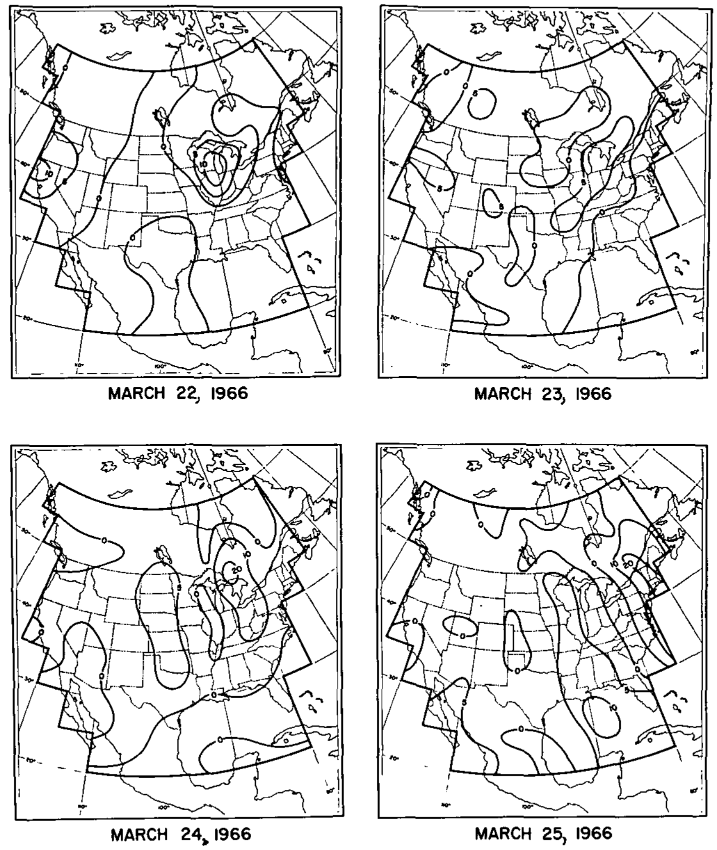


FIGURE 14.—Same as figure 13 for latent heat.

TABLE 5.—*Values of the maximum transports of sensible and latent heat at 700 mb for areas west and east of the 500-mb closed Low center. Units are* $10^2$ *cal* $\cdot s^{-1} \cdot mb^{-1} \cdot cm^{-1}$.

| Date (1966) | Time (GMT) | Sensible heat | | Latent heat | |
|---|---|---|---|---|---|
| | | West of Low center | East of Low center | West of Low center | East of Low center |
| Mar. 22 | 0000 | 23 | 23 | 16 | 12 |
| | 1200 | 30 | 23 | 9 | 11 |
| 23 | 0000 | 34 | 25 | 5 | 7 |
| | 1200 | 45 | 48 | 8 | 31 |
| 24 | 0000 | 41 | 37 | 9 | 21 |
| | 1200 | 40 | 37 | 12 | 34 |
| 25 | 0000 | 34 | 28 | 11 | 25 |

moisture sources of the Gulf of Mexico and the Atlantic Ocean. Table 5 summarizes the maximum transport of sensible and latent heat to the west and east of the Low center during the 4-day period.

An examination of the 500-mb charts (fig. 12) reveals that, as expected, the greatest transport of sensible heat occurred on March 24 when the temperature and pressure fields were most out of phase. It is well known that developing baroclinic disturbances with their out-of-phase relationship combined with an increasing meridional flow produce large transports of heat. Haines and Winston (1963) found that the greatest poleward flux of sensible heat occurs in the regions east of the Asian and North American continents. Both are regions in which there are frequent developments of baroclinic waves. Wiin-Nielsen et al. (1963) found that the ultralong waves (wave no. 1–3) are responsible for a major portion of the poleward heat transport. Wave number 3, for example, is typically associated with troughs off the east coast of North America and Asia. The results obtained in the case studied here suggest that the sensible heat transport is mainly associated with the development of smaller scale baroclinic disturbances that are imbedded in the ultralong waves in these areas.

## 6. SUMMARY

The study of the poleward transport of four forms of energy by four modes of circulation for the winter of 1966 over North America was undertaken to provide estimates of the transport based on an excellent network of radiosonde stations and to permit an examination of the particular contribution the North American sector makes toward the hemispheric transport. The more noteworthy results include:

1. Two computational methods were carried out for the mean meridional circulation. In one method, very large poleward transports of sensible heat and geopotential energy occurred in southern latitudes at high levels. The large values point to large longitudinal variations in the meridional overturnings and suggest a helical structure to the meridional circulation. In the second computational method, vertical mean components were subtracted from the energy terms and the meridional wind component. The results indicated transport values within an order of magnitude of the flux values by other circulation modes. Poleward fluxes occurred in the southern latitudes, where the Hadley circulation is dominant, and weak southward fluxes occurred in middle and northern latitudes.

2. Of the remaining three modes, the transport eddy was dominant. The total energy transport by transient eddies was a maximum near 35°N, in agreement with Oort's (1971*a*) figures for the entire hemisphere. However, over the North American sector for the data period of this study, the maximum flux was nearly 50 percent greater than that for the hemisphere. The latitudinal variation of the transient eddy transport over North America showed a considerably sharper peak at 35°N than for the entire hemisphere indicating that there is a somewhat more restricted latitudinal belt of frequent transient eddies over North America than for the hemisphere as a whole.

3. Energy fluxes calculated by the standing eddy and transient meridional circulation modes amounted to only one-fifth of the fluxes by transient eddies. These two circulation modes were effective in transporting only sensible heat and latent heat, with the sensible heat fluxes largest in high latitudes and latent heat fluxes largest in low latitudes. The transport by the standing eddy mode was less than Oort's (1971*a*) values for the whole hemisphere probably because the large standing eddies are truncated when a limited area is studied.

4. In most previous studies, only sensible heat and geopotential energy were considered; the flux of kinetic energy was considered as negligible. However, in this study it was found that at high levels the transient meridional and standing eddy fluxes of kinetic energy were frequently of the same order as the geopotential energy transport.

5. Wave shape was of definite importance in determining the effectiveness of the poleward transport of energy. Large amplitude troughs with closed Low centers were the most effective transport mechanisms, accomplishing large fluxes of sensible and latent heat at low levels and large fluxes of sensible heat and geopotential energy at high levels. A case study of one closed Low feature revealed, as expected, that the maximum transport occurs during the rapidly intensifying stage of baroclinic disturbances of wave numbers 6–9.


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